ABSTRACT
Diffusivity is one of the most important parameter for modelling drying of stuffed meat products. The knowledge of this parameter is difficult to obtain beforehand as it depends on the characteristics of the product (water content, fat content, salt content, temperature, etc). The aim of this study was to determine the diffusivity coefficients of a commercial type of “salami” using data from its industrial process. Diffusivity coefficients were found by minimizing the error between the weight drying curve obtained from the real process and the curve obtained by simulating the drying process with a mathematical model. Internal mass and heat transfer were modelled using Fick's and Fourier's law respectively. External mass and heat transfer coefficients were calculated using Nusselt adimensional numbers. The drying process was divided into several stages. For each stage a diffusivity value was found. Results showed that diffusivity values lie in the range of values found in previous studies. An exponential equation was found to be the best function to fit the diffusivity data. This function can be reused to simulate the drying of the same product under different process conditions in order to optimize the process in simulation.

Keywords: fermented sausage; modelling; diffusivity; simulation

INTRODUCTION
Simulation has become a standard technique in many industries for process control design. Today, control systems for cars, airplanes, engines, etc., are developed using models prior to testing on real systems. System simulation saves a lot of time and money and in some processes is almost the only procedure used due to the associated risks. Tools such as Matlab-Simulink have allowed engineers in recent years, to easily develop and simulate models of complex systems thus speeding up the process of control design. This is especially important for processes that take place over a long period of time such as drying.

There are no tools to simulate the industrial drying processes of meat products. Simulation can be of great help to design and optimize these processes. These models need to incorporate parameters which must be determined empirically. One of most important parameters is the diffusivity coefficient which governs mass transfer in the product. There are some studies for estimating this parameter in different meat products [1,2,3,4,5,6]. However, the number of these studies is limited and they are usually carried out under controlled environmental conditions (constant temperature, relative humidity and air velocity) and models are simplified. Moreover, the variety of products developed by industry is enormous and these products are processed in changing process conditions, which renders the application of experiments carried out in the laboratories unhelpful.

Therefore, the objective of this work was the development of a software tool based on Matlab-Simulink to determine diffusivity coefficients for stuffed meat products processed in factories which are dried by forced or natural convection and produced in a cylindrical shape. Once the diffusivity coefficients were determined an equation was fitted to them. This equation would make simulation and optimization of the process for the same product for different drying conditions possible.

MATERIALS & METHODS
The software tool for the determination of diffusivity values was developed on Matlab [7] using *.m files. Matlab was used for implementing a model to describe heat and mass transfer for a cylindrical porous moist object subjected to drying. Moreover, a routine was implemented to determine diffusivity values for each process stage from data taken from an industrial process. The following data were necessary for the determination of diffusivity: air temperature (T), air relative humidity (RH), air velocity, weight and time, as well as the initial conditions of the product, i.e weight, moisture, salt content, composition, length and
product diameter. Matlab was also applied to the fitting of an exponential equation to the experimental determined diffusivity values.

To test the software tool, data from an industrial drying process of ‘salami’ was used. Temperature, relative humidity and velocity of air during the drying process were recorded with a datalogger Testo 445 (Testo AG, Lenzkirch, Germany). Air temperature ranged from 12.5 to 13.5 ºC and the relative humidity ranged from 70 to 90%. The average air velocity was 0.2 m/s and the flow of air was continuous. The initial water, fat and salt content was 46.6 %, 33.6 % and 3.2 % respectively. Salami dimensions were 25 cm length (l) and 2.6 cm diameter (d). The duration of the process was 16 days and it was divided into 6 stages according to the changes in air relative humidity (Table 2). The samples initially weighted 0.707 kg and at the end of the process the weight loss was around 22%.

RESULTS & DISCUSSION

Mass and heat transfer were modelled approximating stuffed products to a porous infinite vertical cylinder. Internal mass transfer is governed by Fick’s law of mass diffusion:

\[
\frac{\partial (\rho_m \cdot X)}{\partial t} = \nabla (D \cdot \rho_m \cdot \nabla X)
\]

(1)

where \( t \) is time, \( X \) is the water content (kg water/kg dry matter) and \( \rho_m \) is the dry matter density (kg dry matter/product m\(^3\)), which was kept constant throughout the process. \( D \) is the water diffusivity (m\(^2\)/s).

Internal heat transfer is governed by conduction and it follows a Fourier law of heat conduction.

\[
\rho \cdot c_p \frac{\partial T}{\partial t} = \nabla (k_s \cdot \nabla T)
\]

(2)

where \( T \) is temperature, \( \rho \) is the product density (kg/m\(^3\)), \( c_p \) is the product specific heat (J/kgºC) and \( k_s \) is the product thermal conductivity (W/mK).

Heat and mass transfer are considered to be coupled at the surface. At the surface, mass transferred from the inner part of the product is balanced by evaporation:

\[-\rho_m \cdot D \cdot \nabla X|_{s} = h_m (Y_w - Y_s)\]

(3)

where \( h_m \) is the external mass transfer coefficient (kg dry air/ m\(^2\)s). \( Y_w \) is the moisture content in the bulk air (kg water/kg air), which can be easily calculated from the air bulk temperature and humidity. \( Y_s \) is the moisture content of the air in equilibrium with the product surface and must be calculated from the pork meat sorption isotherms [8].

Heat conducted to the surface from the inner part of the product is balanced by convective and evaporative heat transfer. This equation couples heat and mass transfer.

\[ (k_s \cdot \nabla T)|_{s} = h_t (T_w - T_s) - h_m H_{fg} (Y_w - Y_s) \]

(4)

where \( h_t \) is the convective heat transfer coefficient (W/ m\(^2\)K). \( T_w \) is the temperature in the bulk air and \( T_s \) is the temperature at the product surface. \( H_{fg} \) is the latent heat of evaporation (J/kg).

\( h_m \) and \( h_t \) were calculated using experimental approaches based on the dimensionless Nusselt number for mass and heat transfer. There are two situations, natural convection and forced convection (when cooling coils and ventilation are on).

For natural convection Rayleigh and Grashoff numbers for vertical plates were used. For mass transfer this parameter was calculated as follows:
where $g$ is the gravity (9.81 m/s$^2$), $\rho_b$ is the air density in the bulk and $\rho_s$ is the air density in equilibrium with the product surface. $D_{air}$ is the diffusion coefficient for water diffusing in air and $\mu$ the dynamic viscosity (kg/ms).

and for heat transfer

$$Ra_{heat} = \frac{\beta \cdot g \cdot \text{abs}(T_w - T_r) \cdot L^3}{\nu_i \cdot \alpha}$$

where $\beta$ is the coefficient of thermal expansion (K$^{-1}$), $\nu_i$ is the kinematic viscosity (m$^2$/s) and $\alpha$ is the thermal diffusivity (m$^2$/s).

Heat transfer and mass transfer coefficients were calculated as follows [9]:

$$Nu = \frac{4}{3} \left[ \frac{7 \cdot Gr \cdot Pr^2}{5 \cdot (20 + 21 \cdot Pr)} \right]^{0.25} + \frac{4 \cdot (272 + 315 \cdot Pr) \cdot l}{35 \cdot (64 + 63 \cdot Pr) \cdot d}$$

$Pr$ is the dimensionless Prandtl number which must be replaced by Schmidt number for mass transfer.

For forced convection the external mass transfer coefficient was calculated using the formula proposed by [10] for laminar and turbulent fluxes, as follows:

$$Nu = 0.037 \cdot Pr^{0.43} \left[ Re_L^{0.8} \left( Re_{trans}^{0.8} \cdot Pr^{-0.097} \right)^{0.5} \right]$$

where $Re$ is the Reynolds number. Also in this equation Prandtl number must be replaced by Schmidt number for mass transfer.

As mass and heat transfer are coupled they must be solved simultaneously. In this study, the mass transfer equation was solved before the heat transfer one, which used the mass transferred to the environment as known parameter. This process was iterative and was repeated because mass transfer affects temperature on the surface and vice versa. Obtaining an analytical solution of the governing equations is difficult and therefore approximate methods must be used to solve these equations. Crank-Nicholson finite difference method was selected in this work [11]. The product was discretized to a mesh of 1 mm, which had been proven to be adequate for the proper estimation of $D$. The time step for simulating the process was 10 seconds. As the time steps were small, in most cases no iterations between mass and heat transfer models were necessary.

Table 1 shows the model parameters that were used in this work for simulation. Some of these parameters depend on water content, temperature, etc. However, as a first approximation most of these parameters were kept constant.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>905</td>
</tr>
<tr>
<td>$\rho_m$ (kg dm/m$^3$)</td>
<td>473.3</td>
</tr>
<tr>
<td>$c_p$ (J/kg°C)</td>
<td>2340</td>
</tr>
<tr>
<td>$k_s$ (W/mK)</td>
<td>0.4</td>
</tr>
<tr>
<td>$H_{fg}$ (J/kg)</td>
<td>2502535,259 - 2385.76424 $^\circ$ (Tsurf);</td>
</tr>
</tbody>
</table>

Temperature and water content were considered to be uniform in the product at the beginning of the process.

Diffusivity values were estimated combining the model with data from the real process. (time, temperature, air velocity, air relative humidity and weight). Different diffusivity values were proposed according to the
error between the estimated product weight and the experimental weight at the end of each stage. This error was obtained by simulating the process with the proposed diffusivity value and comparing the weight obtained by simulation with the weight from the process. When the error was below a predetermined value the diffusivity value was considered to be correct for the stage. All this was repeated for each stage of the process. For limiting the search of diffusivity values were limited to a range from 1E-13 ($D_{\text{min}}$) to 1E-9 ($D_{\text{max}}$). First an initial value of 1E-9 ($D_{\text{current}}$) was used which was well above the values found on the subject. Following this the process was simulated for the stage, if the error weight (estimated weight-experimental weight) was below the predetermined value, the search process was stopped and the process was repeated for the next process stage, otherwise a new diffusion value ($D_{\text{new}}$) was tested and calculated, as follows:

\[
\begin{align*}
\text{if}(\text{error weight}>\text{predetermined\_error}) & \\
D_{\text{min}} &= D_{\text{current}}; \\
D_{\text{max}} &= D_{\text{max}}; \\
D_{\text{new}} &= (D_{\text{min}}+D_{\text{max}})/2 \\
\text{elseif}(\text{error weight}<\text{predetermined\_error}) & \\
D_{\text{min}} &= D_{\text{min}}; \\
D_{\text{max}} &= D_{\text{current}} \\
D_{\text{new}} &= (D_{\text{min}}+D_{\text{max}})/2 \\
\text{end}
\end{align*}
\]

After obtaining diffusivity values for each process stage, $D_0$ was calculated using the determined D and temperature at the product surface for each stage, according to this formula, as it follows an Arrhenius equation:

\[
D = D_0 \cdot \exp\left(-\frac{E_a}{RT_s}\right) \quad (9)
\]

$E_a$ is the energy of activation. In [3] the values of $E_a$ were studied for different salt contents. Results showed that this value did not change in a major way with salt content. For this reason $E_a$ was set at 53467 (J/mol). $T$ is the temperature at the product surface.

Table 2 shows data collected from the industrial process considered in this study and the estimated data by the process simulation. Temperature and relative humidity values in the table are the average of the data for each stage. The air velocity was constant, at 0.2 m/s during the whole process.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Time h</th>
<th>T ºC</th>
<th>RH %</th>
<th>Weight g</th>
<th>Experimental data</th>
<th>Estimated data</th>
<th>X surface kgH₂O/kg dm</th>
<th>T surface ºC</th>
<th>D m²/s</th>
<th>D₀ m²/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td>13.68</td>
<td>73.43</td>
<td>662</td>
<td>662.5</td>
<td>0.7834</td>
<td>11.88</td>
<td>2.1883e-10</td>
<td>1.3911</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>13.03</td>
<td>72.05</td>
<td>626.7</td>
<td>627.6</td>
<td>0.6895</td>
<td>11.66</td>
<td>1.4071e-10</td>
<td>0.9103</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>96</td>
<td>12.57</td>
<td>73.69</td>
<td>599</td>
<td>599.3</td>
<td>0.6134</td>
<td>12.01</td>
<td>5.8688e-11</td>
<td>0.3694</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>13.21</td>
<td>69.08</td>
<td>584</td>
<td>584.9</td>
<td>0.5746</td>
<td>12.62</td>
<td>3.9159e-11</td>
<td>0.2348</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>13.49</td>
<td>67.89</td>
<td>577.3</td>
<td>577.9</td>
<td>0.5556</td>
<td>12.91</td>
<td>5.4782e-11</td>
<td>0.3211</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>120</td>
<td>13.38</td>
<td>71.60</td>
<td>553.5</td>
<td>553.9</td>
<td>0.4912</td>
<td>12.99</td>
<td>6.2594e-11</td>
<td>0.3645</td>
<td></td>
</tr>
</tbody>
</table>

The estimated parameters shown in Table 2 are diffusivity coefficients ($D$ and $D_0$), surface water content and temperature for the process under analysis. Estimated diffusivity coefficients were in the range of 3.91E-11 to 2.19E-10 and decreased with the water content at the surface of the product. These results were in line
with values from other studies, but some differences were observed. [2,3,4] studied diffusivity on salted meat products during drying under different process conditions. In these studies, diffusivity ranged from 1.20E-11 to 1.15E-10, but in [2] where the impact of NaCl and temperature on diffusivity was studied, results differed from this work. Diffusivity for a 3% NaCl content (approximately 6.5 kg NaCl/kg H₂O at beginning of the process) and a temperature of 13°C was 2.2E-11 m²/s which was lower than the lowest diffusivity value found in this process (3.91E-11 m²/s). These variations could be explained by the differences in the mathematics used in the model and also by the product texture and composition. Model used by [3] was the analytical solution to the Fick equation under several experimental conditions proposed by [12], which were probably only partially fulfilled. [6] found important differences in diffusivity values when applying [12] with respect to other drying models. Meat texture of the ground stuffed meat used in this study was different to the whole muscle meat used in the reported studies, and there were differences in composition (fat and NaCl content). These variations could also explain the differences in the diffusivity values found in [3] and [4] studies. [13] reported that fat content can affect diffusivity values. [5] studied diffusivity for unsalted pork process) and a temperature of 13ºC was 2.2E-11 m²/s which was lower than the lowest diffusivity value (3.91E-11 m²/s). These variations could be explained by the differences in the mathematics used in the model and also by the product texture and composition. Model used by [3] was the analytical solution to the Fick equation under several experimental conditions proposed by [12], which were probably only partially fulfilled. [6] found important differences in diffusivity values when applying [12] with respect to other drying models. Meat texture of the ground stuffed meat used in this study was different to the whole muscle meat used in the reported studies, and there were differences in composition (fat and NaCl content). These variations could also explain the differences in the diffusivity values found in [3] and [4] studies. [13] reported that fat content can affect diffusivity values. [5] studied diffusivity for unsalted pork

In this case, the resulting function for determining diffusivity was:

$$D = 0.0136 \cdot e^{5.777 \cdot x} \cdot \exp\left(\frac{-53467}{RT_s}\right)$$

(10)

![Figure 1. Estimated points and fitted function for $D_0$ versus estimated water content at the surface](image-url)
CONCLUSION

A software tool was developed on Matlab-Simulink for the determination of diffusivity coefficients affecting the drying of meat products. This tool consisted of a model of mass and heat transfer and was successfully applied to the drying of a "salami" with data obtained from an industrial process.

ACKNOWLEDGMENTS

This work was supported by funds from the reference network in food technology from the Catalan Government (EvalXARTA project 2010 Ref. Ex11) in Spain. The authors gratefully acknowledge the financial participation of the European Community under the Sixth Framework Programme for Research, Technological Development and Demonstration Activities, for the Integrated Project Q-PORKCHAINS FOOD-CT-2007-036245.

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